1 Section 2.5 - 2, 7, 8, 9, 17

1. **2. If $S \subseteq \Re$ is nonempty, show that S is bounded if and only if there exists a closed bounded interval I such that $S \subseteq I$.**

   Proof. ($\Rightarrow$) If S is bounded, then there exists $u = \sup(S)$ and $v = \inf(S)$. Define the interval $I := [v, u]$ and S is contained in I.

   ($\Leftarrow$) If there exists an interval I such that $S \subseteq I = [a, b]$ then $a \leq s$ for all $s \in S$ and likewise, $b \geq s$ for all $s \in S$. Since S is bounded above and below, S, by definition, is bounded. ☐

2. **7. Let $I_n := [0, 1/n]$ for $n \in N$. Prove that $\bigcap_{n=1}^{\infty} I_n = \{0\}$.**

   Proof. It's clear that 0 is in the intersection. The question that comes up is: Is there another number in the set? To prove this, we use contradiction.

   Suppose x is in the intersection, then by 2.4.5 there exists some $k \in N$ so that $1/k < x$. This then means that for all $n \geq k$ we have $1/n < x$. We conclude that x is NOT in any interval I_n for $n \geq k$ or that $x \notin I_n$ for $n \geq k$ contradicting x was in the infinite intersection. ☐

3. **8. Let $J_n := (0, 1/n)$ for $n \in N$. Prove that $\bigcap_{n=1}^{\infty} J_n = \emptyset$.**

   Proof. Since 0 is not in ANY set, it cannot be in the intersection. Again, we use contradiction to assume there is an element in the infinite intersection and just as in Exercise 7, we show that it cannot happen. ☐

4. **9. Let $K_n := (n, \infty)$ for $n \in N$. Prove that $\bigcap_{n=1}^{\infty} K_n = \emptyset$.**

   A little different than the previous proofs, but similar none-the-less.

   Proof. Assume there is an element, x, in the infinite intersection. Then by the Archimedean Property (2.4.3), there exists $k \in N$ such that $x < k$. This means that $x \notin I_k$ for any $n \geq k$. Hence, x couldn't be in the infinite intersection. ☐

5. **17. What rationals are represented by the periodic decimals $1.25137 \cdots 137 \cdots$ and $35.14653 \cdots 653 \cdots$?**

   Using the process outlined in the book (and in algebra classes), set $x = 1.2513 \cdots 137 \cdots$ and then multiply by the appropriate power of 10 to line up the repeating part (1000 in this case), $1000x = 1251.37 \cdots 137 \cdots$. Subtrace the two equations and solve for x. Make sure you multiply by the appropriate number to REMOVE the decimal in the numerator. Solution is $x = 125012/99900$ and $x = 3511139/99900$.