1 Section 3.4 - 4a, 7, 12, 14

1. **4a. Show that $(1-(-1)^n+1/n)$ is divergent.**

   Proof. If you look at the even subsequence (goes to 0) versus the odd subsequence (goes to 2), we have two subsequences that converge to different values. Hence by, 3.4.5 the sequence diverges. □

2. **7. Establish the convergence and find the limits of the following sequences:**

   (a) $\left((1+1/n^2)^{n^2}\right)$

   Proof. Note that we are looking at the subsequence $e_{n^2} \to e$. □

   (b) $((1+1/2n)^n)$

   Proof. $((1+1/2n)^n) = \left((1+1/2n)^{2n*1/n}\right) \to \sqrt{e}$ □

   (c) $\left((1+1/n^2)^{2n^2}\right)$

   Proof. $\left((1+1/n^2)^{2n^2}\right) = \left((1+1/n^2)^{n^2*2}\right) \to e^2$ □

   (d) $((1+2/n)^n)$

   Proof. $((1+2/n)^n) = \left((1+1/(n/2))^{2*n/2}\right) \to e^2$ □

3. **12. Show that if (x_n) is unbounded, then there exists a subsequence (x_{n_k}) such that $\lim 1/x_{n_k} = 0$.**

   Proof. By the Monoton Subsequence Theorem, there exists a subsequence that is either increasing or decreasing. Let's look at the increasing case. This means that $x_{n_1} < x_{n_2} < \dots$. Since the original sequences was unbounded, we know that our subsequence has the same trait. It also implies that $1/x_{n_1} > 1/x_{n_2} > \dots$. Since our our increasing subsequence is getting larger (to Infinity) our limit of the reciprical goes to 0.

   For a decreasing subsequence, we find that our unbounded subsequence is going to -Infinity, but everything else works as intended. □

4. **14. Let (x_n) be a bounded sequence and let $S := \sup\{x_n : n \in N\}$. Show that if $x \notin S$, then there is a subsequence of (x_n) that converges to S.**

   Proof. If $x \notin S$, then we can find $x_{n_1} : x_{n_1} \in (S-1, S)$ and $x_{n_2} : x_{n_2} \in (S-1/2, S)$ where $x_{n_1} < x_{n_2}$ and $x_{n_3} : x_{n_3} \in (S-1/3, S)$ where $x_{n_2} < x_{n_3}$, etc. We have created a subsequence that is increasing and converges to S. □